The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Final Exam

Date: December 10, 2021 Course: EE 313 Evans

Name: ____
Last, First

- This in-person exam is scheduled to last three hours.
- Open books, open notes, and open class materials, including homework assignments and solution sets and previous midterm exams and solutions.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- ***Please disable all wireless connections on your calculator(s) and computer system(s).***
- Please mute all computer systems.
- Please turn off all phones.
- No headphones are allowed.
- All work should be performed on the midterm exam. If more space is needed, then use the backs of the pages.
- **<u>Fully justify your answers</u>.** If you decide to quote text from a source, please give the quote, page number and source citation**.**

Problem	**Point Value**	**Your Score**	**Topic**
1	16		Heart and Soul of Discrete-Time Signals & Systems
2	12		Continuous-Time Fourier Series
3	12		Discrete-Time Audio Signal Processing
4	12		Continuous-Time Communication System
5	12		Discrete-Time Filter Analysis
6	12		Continuous-Time Signal Acrobatics
7	12		Discrete-Time Equalization
8	12		Continuous-Time Frequency-Domain Analysis
Total	100		

Problem 1. Heart and Soul for Discrete-Time Signals and Linear Systems. *16 points.*

(a) **LTI Systems**. Consider a discrete-time linear time-invariant system with input signal $x[n]$, impulse response $h[n]$ and output signal $y[n]$. 9 *points*.

i. Give the relationship for $y[n]$ to $x[n]$ and $h[n]$ involving only operations in the discrete-time domain.

ii. Give the relationship for $Y_{freq}(\hat{\omega})$ to $X_{freq}(\hat{\omega})$ and $H_{freq}(\hat{\omega})$ using only operations in the discrete-time Fourier (frequency) domain.

iii. Give the relationship for $Y(z)$ to $X(z)$ and $H(z)$ using only operations in the z-domain.

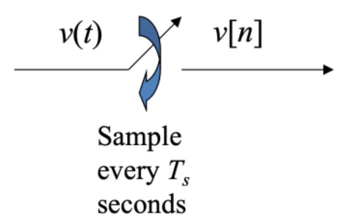
Discrete-time domain	$x[n] \rightarrow$	$\boxed{h[n]}$	$\rightarrow y[n]$	$y[n] =$
Discrete-time Fourier domain	$X_{freq}(\hat{\omega})$	$H_{freq}(\hat{\omega})$	$Y_{freq}(\hat{\omega})$	$Y_{freq}(\hat{\omega}) =$
Z-domain	$X(z)$	$H(z)$	$Y(z)$	$Y(z) =$

(b) **Sampling in the time domain.** *7 points.*

Consider the continuous-time sinusoidal signal $v(t)$ at fixed frequency f_0 in Hz defined as

$$v(t) = \cos(2\pi f_0 t)$$

observed for $-\infty < t < \infty$ and sampled at sampling rate f_s to produce signal $v[n]$ as shown on the right.



i. Give a formula for $v[n]$ observed for $-\infty < n < \infty$. *3 points.*

ii. Give a formula for the discrete-time frequency $\hat{\omega}_0$ for $v[n]$ in rad/sample in terms of the continuous-time frequency f_0. *2 points.*

iii. What continuous-time frequencies f are captured by sampling at sampling rate f_s without aliasing? *2 points.*

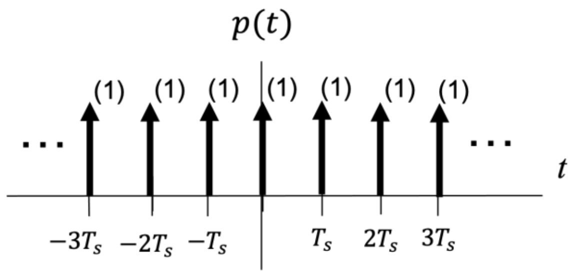
Problem 2. Continuous-Time Fourier Series. *12 points.*

A continuous-time impulse train can model the periodic instantaneous closing and opening of a switch in sampling when viewing the sampling output in continuous time.

For a sampling time of T_s, the impulse train can be expressed as

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\,T_s)$$

and its plot is



where (1) indicates that the area under each Dirac delta is 1.

(a) What is the fundamental period T_0 of $p(t)$? *2 points.*

(b) Compute the Fourier series coefficients using the Fourier synthesis formula. *6 points.*

$$p(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{j\,2\,\pi\,(k\,f_0)\,t}$$

where

$$a_0 = \frac{1}{T_0}\int_{-\frac{1}{2}T_0}^{\frac{1}{2}T_0} p(t)dt$$

$$a_k = \frac{1}{T_0}\int_{-\frac{1}{2}T_0}^{\frac{1}{2}T_0} p(t)\, e^{-j\,2\,\pi\,(k\,f_0)\,t}\,dt$$

The limits of integration are from $-\frac{1}{2}T_0$ to $\frac{1}{2}T_0$ to make sure to include the Dirac delta at the origin inside the limits.

(c) Plot the spectrum of the Fourier series. *2 points.*

(d) Describe the spectrum of the Fourier series coefficients. *2 points.*

Problem 3. Discrete-Time Audio Signal Processing. *12 points.*

(a) Consider generating an A major chord by playing the notes A, C# and E at the same time where the note frequencies are $f_A = 440$ Hz, $f_{C\#} = 550$ Hz and $f_E = 660$ Hz, respectively:

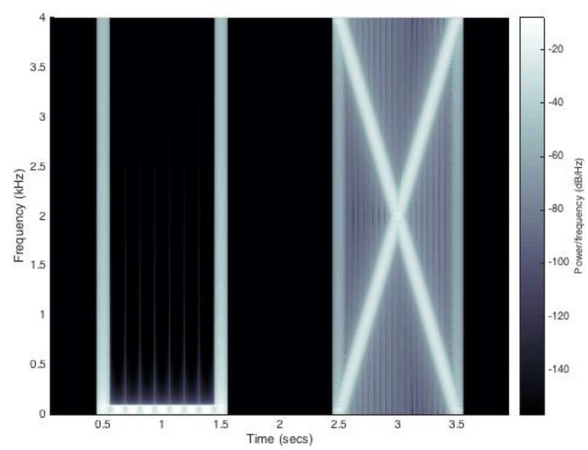
$$x(t) = \cos(2\,\pi\, f_A\, t) + \cos(2\,\pi\, f_{C\#}\, t) + \cos(2\,\pi\, f_E\, t)$$

1. Determine the corresponding discrete-time frequencies $\hat{\omega}_A$, $\hat{\omega}_{C\#}$ and $\hat{\omega}_E$ for a sampling rate of $f_s = 44100$ Hz. *3 points.*

2. What is the smallest discrete-time period in samples for $x[n]$? *3 points.*

(b) A discrete-time signal with sampling rate of f_s of 8000 Hz has the following "UX" spectrogram. The spectrogram was computed using 1000 samples per block and an overlap of 900 samples.

1. Describe the frequency content vs. time. *3 points.*



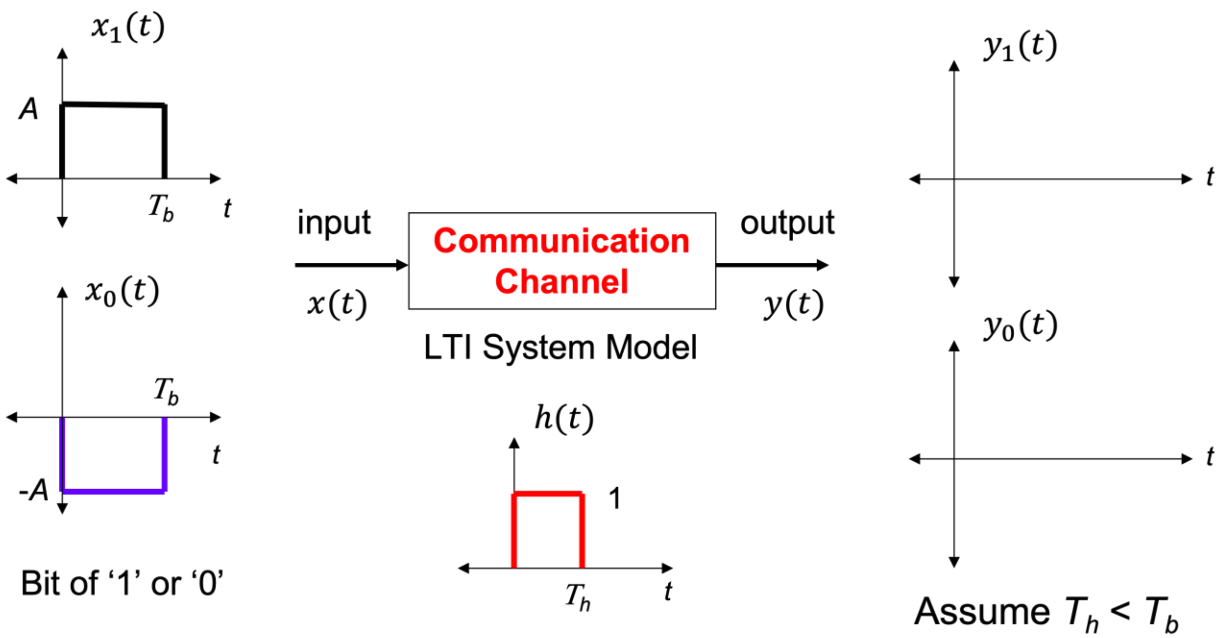
2. What would the signal sound like when played as audio signal? *3 points.*

Problem 4. Continuous-Time Communication System. *12 points*.

We will transmit one bit over a communication channel and analyze the result at the receiver.

- A bit of value '1' will be transmitted as $x_1(t)$, a rectangular pulse of positive amplitude A.
- A bit of value '0' will be transmitted as $x_0(t)$, a rectangular pulse of negative amplitude -A.

We will model the communication channel as an LTI system, as given below.



(a) Plot $y_1(t) = h(t) * x_1(t)$. Label the important points on the vertical and horizontal axes in terms of A, T_b, and T_h. Assume $T_h < T_b$. *4 points.*

(b) Plot $y_0(t) = h(t) * x_0(t)$. Label the important points on the vertical and horizontal axes in terms of A, T_b, and T_h. Assume $T_h < T_b$. *4 points.*

(c) Determine how the receiver could reliably determine which bit had been transmitted by processing $y(t)$. *4 points.*

Problem 5. Discrete-Time Filter Analysis. *12 points.*

Consider the following causal finite impulse response (FIR) linear time-invariant (LTI) filter with input $x[n]$ and output $y[n]$ described by

$$y[n] = x[n] - x[n-2]$$

for $n \geq 0$.

(a) What are the initial conditions? What are their values? *3 points*.

(b) Derive the system transfer function $H(z)$ in the z-domain and the region of convergence. *3 points*

(c) Give a formula for the discrete-time frequency response of the FIR filter. *3 points*.

(d) What is the frequency selectivity: lowpass, highpass, bandpass, bandstop, allpass, notch? *3 points.*

Problem 6. Continuous-Time Signal Acrobatics. *12 points.*

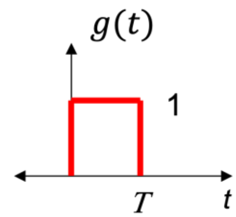
Matched filtering detects a pulse shape in a signal by correlating the signal with the known pulse shape. Applications include communication, radar, sonar, and ultrasound systems.

The matched filter gets its name from its impulse response $h(t)$ being matched to the pulse shape $g(t)$ according to the following formula:

$$h(t) = C\, g^*(T - t)$$

We form $h(t)$ by flipping $g(t)$ in time t, delaying by constant delay T, conjugating the amplitude, and scaling by non-zero constant C. T is often chosen to make the impulse response causal.

For the pulse shape $g(t)$ shown on the right,



(a) Plot $g(-t)$. *4 points.*

(b) Plot $g(T - t)$. This should be a causal signal. *4 points.*

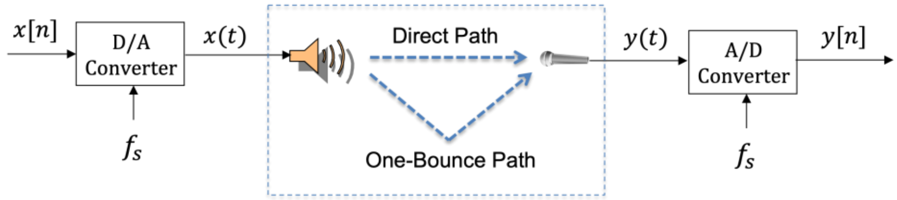
(c) Plot $C\, g^*(T - t)$. *4 points.*

Problem 7. Discrete-Time Equalization. *12 points.*

When sound waves propagate through air, or when electromagnetic waves propagate through air, the waves are absorbed, reflected and scattered by objects in the environment.

In the transmission of sound waves over the air in a room from an audio speaker to a microphone, we will model the direct path from the speaker to the microphone as having zero delay, and a one-bounce path from the speaker to an object and then to the microphone having delay t_1.

This single reflection is a type of echo.



We model the signal $y(t)$ at the output of the microphone as

$$y(t) = x(t) - \alpha\, x(t - t_1)$$

where α is a real-valued constant and $t_1 > 0$.

We model that system that connects $x(t)$ and $y(t)$ as linear and time-invariant (LTI).

By adding a digital-to-analog (D/A) converter on the input the audio speaker and an analog-to-digital converter (A/D) on the output of the microphone, we convert the problem to discrete time:

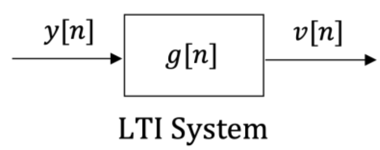
$$y[n] = x[n] - \alpha\, x[n-1]$$

We're assuming $t_1 = T_s$, α is real-valued, and delay through the D/A and A/D converters is zero.

(a) Derive a formula for the impulse response $h[n]$. *3 points.*

(b) Find transfer function in the *z*-domain $H(z)$. *3 points.*

(c) We add a discrete-time LTI filter at the microphone output to remove as much echo as possible. Design the discrete-time filter by giving its transfer function $G(z)$ in the *z*-domain. The filter $G(z)$ must be bounded-input bounded-output (BIBO) stable. *6 points.*



- Case I. $|\alpha| < 1$.

- Case II. $|\alpha| = 1$.

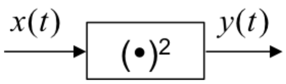
- Case III. $|\alpha| > 1$.

Problem 8. Continuous-Time Frequency-Domain Analysis. *12 points.*

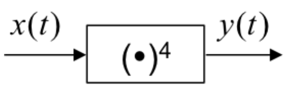
For each problem below, determine the frequency (or frequencies) present in $x(t)$ and $y(t)$ as well as the single sampling rate you would use for the entire system to prevent aliasing.

Please note that $T_c = 1 \,/\, f_c$ and $T_0 = 1 \,/\, f_0$ in the following. *Each part is worth 4 points.*

(c) Let $x(t) = \cos(2\pi f_c t)$ be a continuous-time signal for $-\infty < t < \infty$.



(d) Let $x(t) = \cos(2\pi f_c t)$ be a continuous-time signal for $-\infty < t < \infty$.



(e) Let

$$x(t) = \operatorname{sinc}\left(\frac{t}{T_0}\right)$$

be a continuous-time signal for $-\infty < t < \infty$

whose continuous-time Fourier transform is

$$X(f) = T_0 \operatorname{rect}\left(\frac{f}{f_0}\right)$$

Here, $f_c > f_0$